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Generalized Teleportation of a d-Level N-Particle GHZ State with One Pair of Entangled Particles as the Quantum Channel

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Abstract With one pair of entangled particles as the quantum channel, we present an explicit generalized protocol for perfectly teleporting a d-level N-particle GHZ state from a sender to a receiver. This protocol has the advantage of transmitting much less particles and classical information for teleporting the d-level N-particle GHZ state than others.

Keywords Generalized teleportation · d-Level N-particle GHZ state · CNOT operation

1 Introduction

The concept of entanglement is central in quantum information processing. One major breakthrough was obtained by Bennett *et al.* [1], who created the quantum teleportation protocol. Quantum teleportation, a unique thing in quantum mechanics, has no counterpart in classical physics. This technique allows to remote parties, say the sender (Alice) and the receiver (Bob), to exploit the nonlocal correlation of the quantum channel shared initially to teleport an unknown quantum state $|\kappa\rangle = a|0\rangle + b|1\rangle$ from a place to another one by prearranging the share of an Einstein-Podolsky-Rosen (EPR) state and some classical information. To this end, the sender Alice makes a joint Bell-state measurement on the unknown quantum system and her EPR particle, and Bob can recover the unknown quantum state in his place according to the relationship between the Alice's measurement results and the state of his EPR particle. Quantum teleportation has received much attention [1–10] both theoretically and experimentally in recent years due to its important applications in quantum

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communication. For example, in 1998, Karlsson *et al.* [2] generalized Bennett *et al.*'s teleportation idea by using a 3-qubit Greenberger-Horne-Zeilinger (GHZ) state $|000\rangle + |111\rangle$ [11] instead of an EPR pair. The teleportation of an arbitrary two-qubit state had been studied by Lee *et al.* [3] and recently by Rigolin [7].

All the previous proposals assume, nevertheless, that the quantum channels used to teleport the qubits are noiseless maximally entangled states. But in a realistic situation, however, decoherence and noise degrade the channel and we do not have a maximally entangled state anymore. To overcome this flaw, there are four approaches [12] to teleportation protocol that the senders and the receiver can take: (1) One way out of this problem is to employ quantum distillation protocol [13], which allow us to obtain a maximally entangled state from a large ensemble of partially states. But quantum distillation only achieves a maximally entangled state asymptotically. Thus, for finite runs of the distillation protocol we always obtain an almost maximally entangled state. (2) The sender may perform the standard Bell-state measurements directly with the nonmaximally entangled states and let the receiver apply an auxiliary unitary transformation with the aid of an auxiliary qubit to obtain the desired state probabilistically [14, 15]. The success probability is less than 1, but the receiver knows whether he(she) obtained the desired state or not by observing the state of the auxiliary qubit. (3) Alice may perform "generalized measurements" [16–20] upon the nonmaximally entangled pairs that distinguish nonorthogonal "Bell-type" states conclusively with a certain probability less than 1. For example, Xia *et al.* [20] present a reexamining generalized protocol for probabilistic teleportation of an arbitrary N-qubit GHZ entangled state via only one non-maximally two qubit entangled state. Gordon *et al.* [17, 18] present an interesting generalized teleportation protocol where the channels are not maximally entangled states. In this protocol, by properly choosing the measurement basis it is possible to achieve unity fidelity transfer of the state.

In this paper, we consider a generalized quantum teleportation protocol for an arbitrary d-level N-particle GHZ entangled state [21] using only one two-particle entangled state channel and some operation with the aid of classical information. The obvious advantage of the protocol is that the quantum channel is only one two-particle entangled state, and the two parties need not transmit many particles for setting up the quantum channel, which will reduce largely the entangled quantum resource. In a noise channel, this protocol is obviously very convenient and practical. Because the d-level N-particle GHZ-like state has only d coefficients and behaves like a d-level particle, the receiver constructed from a single particle state by concatenating additional particles to it using a initialization algorithm, the circuit is just like a state initialization as in [22].

The paper is organized as follows. In Sect. 2, we first review the protocol for teleportation of a 2-level N-qubit GHZ entangled state via only one two-qubit entangled state. In Sect. 3, we generalize the protocol to teleport a d-level N-particle GHZ-type entangled state using one d-level two-particle entangled state. Discussions and conclusions appear in Sect. 4.

2 Quantum Teleportation of a 2-Level N-Particle GHZ Entangled State by Only One d-Level Entanglement Two-Particle State

For the sake of the clearness, let us first review Xia *et al.*'s [17, 18] generalized teleportation protocol. In this paper, the state that Alice wants to teleport is the pure GHZ-type state for N qubits, $|\psi^T\rangle = \sum_{i=0}^{1} a_i |i\rangle_{1,2,\ldots,N}^{\otimes N}$, with a_i complex and $\sum_{i=0}^{1} |a_i|^2 = 1$. The channel can be constructed as follows. Alice shares with Bob the state $|\phi\rangle^{AB} = N(|00\rangle_{AB} + n|11\rangle_{AB})$, where n can be complex and $N = 1/\sqrt{1+|n|^2}$. The first qubit A belongs to Alice, the second one B to Bob. Note that here we allow n to be any complex number and only for $n = 1$ we recover the Hillery *et al.* channel. The concurrence for this state, a well-known entanglement monotone [23], is $c(n) = 2|n|/(1 + |n|^2)$, which is a monotonically increasing function of $|n|$. (a) Alice performs a generalized Bell measurement on qubits A and 1. (b) Alice transmits the acceptable results (2 bits) of her measurement via a classical channel to Bob, who has 16 free parameters, four for each of Alice's measurement results. Bob performs unitary operations like [17, 18] on his qubits according to the classical information received from Alice. We restrict ourselves, however, to only one free parameter (θ_j) for each result. The unitary operation are $\{R_j\} \to \exp(i\sigma_z\theta_j)O_j$, where $\{O_j\} = \{I, \sigma_z, \sigma_x, \sigma_z\sigma_x\}$. I is the identity and σ are the usual Pauli matrices. Then the unitary operations above transfer the state of the particles of $2, 3, \ldots, N$ and B into the following state: $|\psi\rangle_{2,3,\ldots,N} = a_0|0\rangle^{N-1}_{2,3,\ldots,N}|0\rangle_B + a_1|1\rangle^{N-1}_{2,3,\ldots,N}|1\rangle_B$, with the $C_N^{pro} = \frac{2}{2^N+1}(\sum_{i=1}^{N} 2^{i-1}P_i^N)$, where P_i^N is the sum of all permutations of th product of i variables out of all $\{\chi_r\}$, where $r = 1, \ldots, N$, and $\chi_r = c(n_r)c(m_r)/2$. (c) Alice takes a Hadamard operation on particle 2 and makes a von Neumann measurement on particle 2 in the basis {0, 1}. Then Alice repeats this process for the other particles $3, 4, \ldots, N$, and informs Bob of the measurement results. (d) Bob performs a single qubit operation I or σ_z, conditioned on Alice's classical information, on his particle B which will be transformed into $|\psi\rangle_B = a_0|0\rangle_B + a_1|1\rangle_B$. (e) Bob introduces $N - 1$ ancillary particles $b_1, b_2, \ldots, b_{n-1}$ in the initial state $|00\cdots 0\rangle_{b_1b_2\cdots b_{N-1}}$ and entangle them with the non-maximally two-qubit entangled state B, see Fig. 2. That is, Bob takes CNOT operation on the qubit B and an auxiliary particle b_l ($l = 1, 2, \ldots, N - 1$) by using the qubit B as the control qubit. After all these CONT operations, the state of the composite quantum system composed of the qubit B and b_l becomes $|\psi\rangle_{Bb_1b_2\cdots b_{N-1}} = (a_0|0\rangle^{\otimes N} + a_1|1\rangle^{\otimes N})_{Bb_1b_2\cdots b_{N-1}}$, this is the state that Alice wants to send to Bob. The probabilistic quantum teleportation of arbitrary N-qubit GHZ entangled state's protocol efficiency are $P_{suc} = 2n^2/(1 + n^2)^2$.

3 Quantum Teleportation of a *d*-Level *N*-Particle GHZ Entangled State by Only One *d*-Level Entanglement Two-Particle State

In this section we will generalize the above protocol to arbitrarily higher level which are referred to specifically as d-level here.

Suppose that the state Alice wants to send to Bob is

$$|\psi^T\rangle = \sum_j e^{i\delta_j}|j\rangle^{\otimes N}_{1,2,\ldots,N}/\sqrt{d}, \tag{1}$$

where $\delta_0 = 0$. We define the above state $|\psi^T\rangle$ as an equatorial entangled state in this paper since the components of the state have the same norm. In order to realize teleportation, the sender, Alice, first shares an entangled particles pair $|\phi\rangle^{AB} = \sum_l |ll\rangle/\sqrt{d}$ with the receiver, Bob, where $j, l = 0, 1, \ldots, d - 1$. The principle for determining the relation between the state of the particle u_1 and the others u_x ($x = 2, 3, \ldots, N$) is shown in Fig. 1. In detail, Alice takes $d - 1$ d-level CNOT operations on the particle u_1 and the particle u_2 by using u_1 as the control particle. After this d-level CNOT operation, the state of $|\psi^T\rangle$ becomes

$$|\psi^T\rangle = \sum_j e^{i\delta_j}|j\rangle^{\otimes N-1}_{1,3,\ldots,N}/\sqrt{d} \otimes |(\oplus^{d-1}J_1) \oplus J_2\rangle, \tag{2}$$

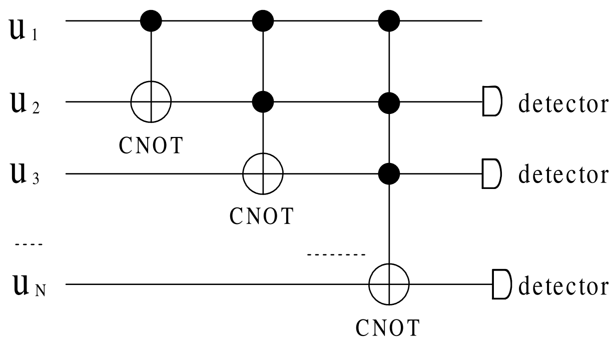


Fig. 1 The schematic principle for determining the relation between the state of the particle u_1 and the others u_x $(x = 2, 3, \dots, N)$

where the $\oplus$ denotes an addition mod d. Alice repeats this process for the other particles u_x $(x = 3, 4, \dots, N)$ and obtains the state of the unknown quantum system as

$$|\psi^T\rangle = \sum_j e^{i\delta_j}|j\rangle/\sqrt{d} \otimes^{N-1} |(\oplus^{d-1}J_1) \oplus J_x\rangle. \tag{3}$$

Alice measures the particles u_x $(x = 2, 3, \dots, N)$ with the MB $\{|0\rangle, |1\rangle, \dots, |d-1\rangle\}$ and obtains the relation $(\oplus^{d-1}J_1) \oplus J_x$. After all the $N-1$ d-level CNOT operations and single-particle measurements, the state of the particle u_1 is transformed into

$$|\psi^{T_1}\rangle = \sum_j e^{i\delta_j}|j\rangle/\sqrt{d}. \tag{4}$$

Now Alice performs a two-particle projective measurement on her two particles (1, A) in the following basis vectors (see Fig. 2)

$$|\phi\rangle_{nm}^{1A} = \sum_j e^{2\pi ijn/d} e^{-i\delta_j}|j\rangle \otimes |j + m \bmod d\rangle/\sqrt{d}. \tag{5}$$

After the measurement, the particle (B) will be

$$|\psi\rangle_{nm}^{B} = \sum_j e^{-2\pi ijn/d} e^{i\delta_j}|j + m \bmod d\rangle/\sqrt{d}. \tag{6}$$

According to Alice's information, Bob carries out an appropriate local unitary operation [10] and fulfill the following transformation

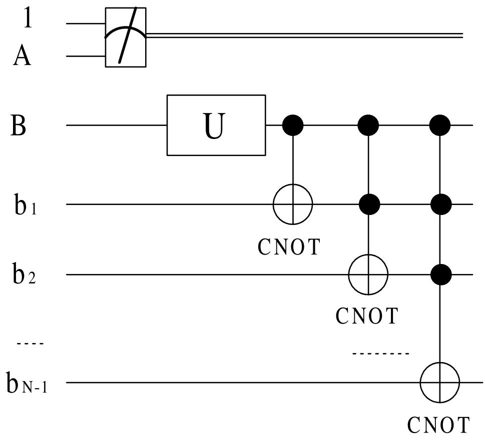
$$|\psi\rangle_{00}^{B} = U_{nm}|\psi\rangle_{nm}^{B} = |\psi^{T_1}\rangle, \tag{7}$$

where

$$U_{nm} = e^{2\pi jn/d}\delta_{j+m \bmod d}. \tag{8}$$

Bob should first prepare $N-1$ d-level auxiliary particles $b_1, b_2, \dots,$ and b_N in the state $|000\dots0\rangle_{b_1b_2\dots b_{n-1}}$, and entangle them with B. That is, Bob takes $N-1$ d-level CNOT operations [22] $(\oplus^{d-1}J_B \oplus 0_{b_x}$, where the $\oplus$ denotes an addition mod $d)$ on the particle B and an auxiliary particles b_x $(x = 1, 2, \dots, N)$ by using the particle B as the control particle (see Fig. 2). After all the d-level CNOT operations, the state of the composite quantum system composed of the particles B and b_x become $|\psi^T\rangle$.

Fig. 2 The schematic principle for reconstructing the d-level N-particle GHZ entangled state



If the state Alice wants to teleport is $|\psi^T\rangle = \sum_{j=1}^{d} c_j e^{i\delta_j} |j\rangle_{1,2,\ldots,N}^{\otimes N}$ and the quantum channel is $|\phi\rangle^{AB} = \sum_{l=1}^{d} a_i |ll\rangle_{AB}$, then the state $|\psi^T\rangle$, with certain probability, can be teleported by employing a positive operator valued measurement [24] $\hat{M}'_\alpha = \lambda' |\hat{\psi}'_\alpha\rangle\langle\hat{\psi}'_\alpha|$ as the joint measurement, we can achieve the protocol for teleporting the equatorial entangled state following the aforementioned method. It is apparent that under such circumstances our generalized teleportation protocol can be accomplished probabilistically.

4 Conclusion

In summary, we have presented a protocol for teleporting an d-level N-particle GHZ state between two parties (Alice and Bob) using only one d-level two-particle quantum system as the quantum channel. We found that the generalization protocol for d-level N-particle GHZ state can be perfectly realized whatsoever the level d is. But how to generalize this protocol to teleport a d-level N-particle GHZ state in real Hilbert space needs further investigation. Compared with previous protocols, what deserves mentioning here is that, using $2(N-1)$ d-level CNOT operations, our protocol can be generalized to teleport a d-level N-particle GHZ state by means of only one entangled pair. The obvious advantage of the protocol is that the two parties do not need to transmit many particles for setting up the quantum channel, which will reduce largely the entangled quantum resource. In a noise channel, this protocol is obviously very convenient and practical. We hope that this new protocol can be useful for further study of quantum teleportation.

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